

# Sequentially complete markets remain incomplete

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# Sequentially Complete Markets Remain Incomplete<sup>1</sup>

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## **Abstract**

We reconsider the well-known result of Arrow (1953) that the set of equilibria of an economy with complete markets coincides with the one of an economy with sequentially complete markets. We show by means of two examples that this result is problematic when there exist multiple equilibrium continuations to the initial-period component of an intertemporal equilibrium. Some consequences are drawn.

*Keywords:* *Sequentially Complete Markets, Rational Expectations, Time Inconsistency*

*JEL classification:* *D52, D61*

## Introduction

In the standard Arrow-Debreu general equilibrium model, uncertainty is captured by an event tree, each path of which describes a potential future history of the fundamentals of the economy. This representation was introduced in Arrow (1953), where two results are presented: (i) Every efficient allocation can be sustained as a competitive equilibrium in markets for contingent claims to commodities (Theorem 1). (ii) Every efficient allocation can be sustained as a competitive equilibrium in markets for elementary securities and in spot markets for commodities (Theorem 2). An elementary security pays one unit of numeraire if and only if a specific event obtains. Markets are sequentially complete if, at each date event, there exist markets for elementary securities contingent on every immediate successor to the event.

Arrow Theorem 2 is a “possibility theorem” (for a change...) that does not refer to the expectations held initially about the allocations due to emerge at future date events on competitive spot markets there. Indeed, expectations were not a daily concern in 1953, 8 years before the seminal paper by Muth (1961). It is only with the work of Radner (1972) that a sharper statement of Arrow’s possibility theorem became possible: *every efficient allocation can be sustained as “an equilibrium of plans, prices and price expectations on a sequence of markets”*.

The equilibrium concept used by Radner (1972) imposes that expectations held initially by all agents, about the prices that will prevail on the markets open at a future date event, be (i) *correct*, in the sense that these are indeed market-clearing prices; (ii) *common* to all agents; and (iii) *single valued*. Most authors - including e.g. Dutta and Morris (1997) - refer to these three requirements as defining a *rational expectations equilibrium*, REE. We adopt that terminology here.

The condition that price expectations be single valued is problematic when there exist multiple competitive equilibria on the spot markets at future date events. Weaker requirements then define alternatives worthy of attention. Dutta and Morris (1997) define a *common beliefs equilibrium*, CBE, by the single condition that (iv) all agents assign common probabilities to every el-

ement of the set of competitive equilibria on markets at a future date event, and these probabilities add up to one. Clearly, an REE is a special case of CBE with all the probability concentrated on a single future equilibrium.

These authors provide an example of an economy with asymmetric information which owns a single REE but a continuum of CBE's.

Under strict convexity of preferences, a CBE assigning positive probability to several equilibria entailing different allocations cannot be efficient: the probability mixture is dominated by convex combinations of the different allocations (for instance, with weights equal to the respective probabilities). Thus, CBE's that are not REE's (that are not single valued) are inefficient, and their existence raises an issue about the significance of Arrow's Theorem 2. The following example illustrates.

### An Example

Our example is borrowed from Drèze (1999), where it is offered as an illustration of some work by Chichilnisky (1999) and Hahn (1999). It uses the simplest possible framework, namely a two-period exchange economy with two goods, two agents and no uncertainty. Markets for contingent commodities thus reduce to forward markets. Two market structures are contrasted: (1) period 0 markets for the exchange of the two spot commodities and the two forward commodities; (2) period 0 markets for the exchange of the two spot commodities and a nominal bond, followed by period 1 markets for the exchange there of the two spot commodities and the numeraire. (Our normalization plus the symmetry of the example make the numeraire equivalent to a composite commodity consisting of one unit each of the two physical commodities.)

Agents are  $i$  and  $j$ . Commodities are 01 and 02 at period 0, 11 and 12 at period 1. The endowments are  $w^i = (2, 2, 0, 4)$  and  $w^j = (2, 2, 4, 0)$ . Consumption vectors are

$$x^h = (x_{01}^h, x_{02}^h, x_{11}^h, x_{12}^h), \quad h = i, j.$$

The preferences of  $i$  and  $j$  are represented by the following utility functions:

$$u^i = v^i(x_{01}^i, x_{02}^i)^{\frac{1}{2}} \cdot v^i(x_{11}^i, x_{12}^i)^{\frac{1}{2}}, \text{ where}$$

$$v^i(x_{\cdot 1}^i, x_{\cdot 2}^i) = \begin{cases} [\min(x_{\cdot 1}^i, x_{\cdot 2}^i)], & \min(x_{\cdot 1}^i, x_{\cdot 2}^i) \leq 1, \\ [(x_{\cdot 1}^i - 1)^{\frac{1}{2}}(x_{\cdot 2}^i - 1)^{\frac{1}{2}} + 1], & \min(x_{\cdot 1}^i, x_{\cdot 2}^i) \geq 1, \end{cases}$$

$u^j = v^j(x_{01}^j, x_{02}^j)^{\frac{1}{2}} \cdot v^j(x_{11}^j, x_{12}^j)^{\frac{1}{2}}$ , where

$$v^j(x_{\cdot 1}^j, x_{\cdot 2}^j) = \begin{cases} [\min(x_{\cdot 1}^j, x_{\cdot 2}^j)], & \min(x_{\cdot 1}^j, x_{\cdot 2}^j) \leq 3, \\ [(x_{\cdot 1}^j - 3)^{\frac{1}{2}}(x_{\cdot 2}^j - 3)^{\frac{1}{2}} + 3], & \min(x_{\cdot 1}^j, x_{\cdot 2}^j) \geq 3. \end{cases}$$

The Edgeworth box in Figure 1 depicts the preferences of the two agents in either period. Aggregate endowment, (4,4), is at point  $E_a$  in period 0, at point  $w$  in period 1.

It may be verified (see Appendix 1) that there exists a single competitive equilibrium on the markets at period 0 for spot and forward commodities. When we normalize prices to sum up to two, the competitive equilibrium is given by prices  $p^* = (1/2, 1/2, 1/2, 1/2)$  and allocation  $x^i = x^j = (2, 2, 2, 2)$ , corresponding to point  $E_a$  in both periods. Using equality of marginal rates of substitution and price ratios, as well as the properties of the minimum functions, it is easily verified that this is a competitive equilibrium indeed. The net trades are  $z^i = (0, 0, 2, -2)$  and  $z^j = (0, 0, -2, 2)$ .

The same allocation is sustained by no trade at period 0 in either spot commodities or the nominal bonds, followed by net trade at period 1,  $z_1^i = (2, -2)$ ,  $z_1^j = (-2, 2)$ , at the competitive spot prices  $p_1^* = (1/2, 1/2)$ . That sequence defines a REE, the unique REE in this example. However - and *this is the whole point of the example* - there exists at period 1 another competitive equilibrium for the exchange economy with initial endowments  $w_1^i = (0, 4)$ ,  $w_1^j = (4, 0)$ , namely point  $E_b$  in Figure 1 with  $z_1^i = (-3, 1)$ ,  $z_1^j = (3, -1)$ , and prices  $\hat{p}_1 = (3/4, 1/4)$ . Although the allocation  $\hat{x}^i = (2, 2, 1, 1)$ ,  $\hat{x}^j = (2, 2, 3, 3)$  is not sustained as an equilibrium at period 0 in spot and forward commodities, it could result from no trade in the spot-and-bond markets at period 0 (as called for by the unique REE), followed by the competitive equilibrium on

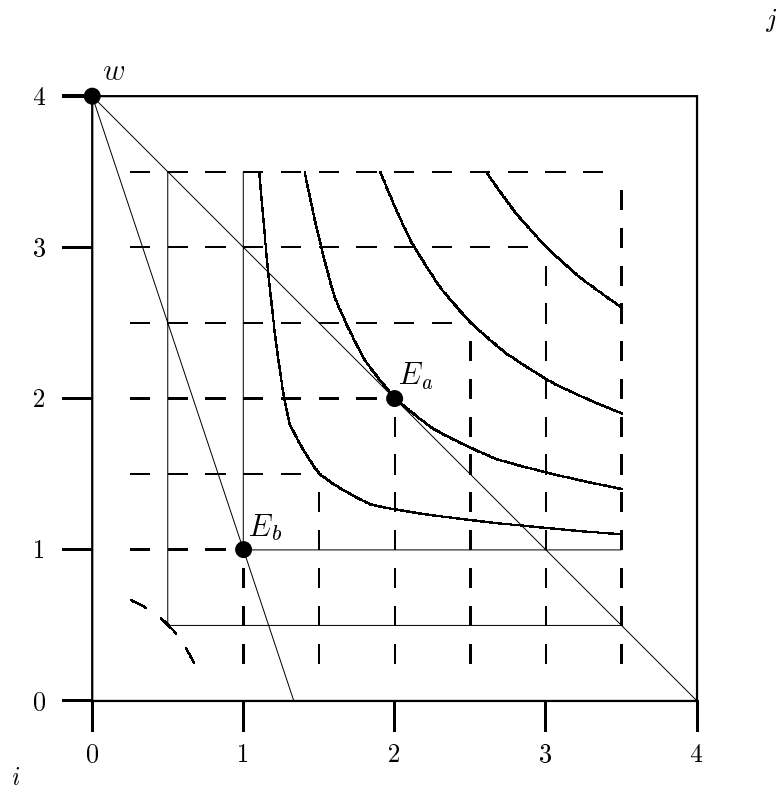


Figure 1: Multiple equilibria at period 1. The indifference curves of  $i$  are solid lines, those of  $j$  are dashed.

spot markets  $E_b$  at period 1. Notice that this allocation is not Pareto efficient. It is Pareto dominated by the allocation  $x_\varepsilon^i = (2 - \varepsilon, 2 - \varepsilon, 1 + \varepsilon, 1 + \varepsilon)$ ,  $x_\varepsilon^j = (2 + \varepsilon, 2 + \varepsilon, 3 - \varepsilon, 3 - \varepsilon)$ , for  $0 < \varepsilon < 1$ .

The existence at period 1 of several competitive equilibria compatible with no trade at period 0 raises the question: on what grounds is one of them privileged as a part of the REE? If rationality is understood, as in Muth (1961), to include knowledge of the model, then existence of multiple period 1 equilibria is common knowledge, and should be taken into account by the agents; or else, one should specify an equilibrium selection mechanism, and verify that it implements the REE.

Before leaving our example, it should be noted that it admits a continuum of CBE's, indexed by the probabilities of  $E_b$  (with suitable spot prices) and some  $E'_a$  (with prices  $(1/2, 1/2)$ ) on the main diagonal to the right of  $E_a$ . Indeed, the prospect of period 1 prices with  $p_{11} > p_{12}$  supporting  $E_b$  leads  $i$  to demand, and  $j$  to supply bonds - thereby displacing the endowment point and shifting  $E_a$  to the right if the period 1 prices are  $(1/2, 1/2)$ . For instance, under equal probabilities for  $E_b$  and  $E'_a$ , there exists (as verified in Appendix 2) a CBE with  $x_0^i = 1.63$ ,  $x_0^j = 2.37$ , bond price  $q = 1.064$ ,  $p_0 = (\frac{1}{2}, \frac{1}{2})$ ,  $p_1 = (\frac{1}{2}, \frac{1}{2})$  with probability 0.5 and  $p_1 = (0.837, 0.163)$  with probability 0.5.

The period 1 equilibrium is either  $E'_a$  with  $x_1^i = 2.37$ ,  $x_1^j = 1.63$ , or  $E_b$ . Alternative probabilities for the two equilibria result in different values of the initial bond trades, future spot prices at  $E_b$  and allocation at  $E'_a$ .



## Comments

1. The example is constructed to be simple and transparent. Preferences satisfy continuity, but not differentiability. It should be obvious that a robust example with differentiable preferences is at hand.
2. The example illustrates the existence, under sequentially complete markets, of a multiplicity of “common beliefs equilibria,” only one of which is efficient (is an REE). In the example, the efficient equilibrium entails common single-valued expectations about the last-period equilibrium. But there exists another last-period equilibrium consistent with the initial-period trades. The rationality of the single-valued expectations is thus problematic, pending a further specification of the selection mechanism for the last-period equilibrium. Equilibrium theory does not consider such a mechanism.<sup>1</sup>
3. A simple selection mechanism is Walrasian tâtonnement, under a given starting point. In our example, using the period spot prices as starting point, equilibrium  $E_a$  is obtained at once. However, it is easy to produce examples where unchanged prices (between periods 0 and 1) selects the wrong equilibrium. Suppose that

$$u^i = v^i(x_{01}^i, x_{02}^i)^{\frac{1}{3}} \cdot v^i(x_{11}^i, x_{12}^i)^{\frac{2}{3}}, \text{ where}$$

$$\begin{aligned} v^i(x_{01}^i, x_{02}^i) &= (x_{01}^i)^{\frac{3}{4}}(x_{02}^i)^{\frac{1}{4}} \\ v^i(x_{11}^i, x_{12}^i) &= \begin{cases} [\min(x_{11}^i, x_{12}^i)], & \min(x_{11}^i, x_{12}^i) \leq 1, \\ [(x_{11}^i - 1)^{\frac{1}{2}}(x_{12}^i - 1)^{\frac{1}{2}} + 1], & \min(x_{11}^i, x_{12}^i) \geq 1, \end{cases} \end{aligned}$$

$$u^j = v^j(x_{01}^j, x_{02}^j)^{\frac{3}{5}} \cdot v^j(x_{11}^j, x_{12}^j)^{\frac{2}{5}}, \text{ where}$$

$$\begin{aligned} v^j(x_{01}^j, x_{02}^j) &= (x_{01}^j)^{\frac{3}{4}}(x_{02}^j)^{\frac{1}{4}} \\ v^j(x_{11}^j, x_{12}^j) &= \begin{cases} [\min(x_{11}^j, x_{12}^j)], & \min(x_{11}^j, x_{12}^j) \leq 3, \\ [(x_{11}^j - 3)^{\frac{1}{2}}(x_{12}^j - 3)^{\frac{1}{2}} + 3], & \min(x_{11}^j, x_{12}^j) \geq 3. \end{cases} \end{aligned}$$

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<sup>1</sup>Formal treatment of such situations, as in Cass and Shell (1983) or Chichilnisky (1999), relies on a random-selection mechanism.

Consider initial endowments given by  $w^i = (1, 1, 0, 4)$  and  $w^j = (3, 3, 4, 0)$ . One obtains a competitive equilibrium on the markets at period 0 for spot and forward commodities, with prices  $p^* = (3/4, 1/4, 1/2, 1/2)$  and allocation  $x^i = (1, 1, 2, 2)$ ,  $x^j = (3, 3, 2, 2)$ . The net trades are  $z^i = (0, 0, 2, -2)$ ,  $z^j = (0, 0, -2, 2)$ . The same allocation is sustained by no trade at period 0 in either spot commodities or the nominal bond, followed by net trades at period 1,  $z_1^i = (2, -2)$ ,  $z_1^j = (-2, 2)$  at the competitive spot prices  $p_1^* = (1/2, 1/2)$ . However, as before, there exists at period 1 another competitive equilibrium for the exchange economy with initial endowments  $w^i = (0, 4)$ ,  $w^j = (4, 0)$ . It is defined by the prices  $\hat{p}_1 = (3/4, 1/4)$  and the net trades  $z_j^i = (1, -3)$ ,  $z_1^j = (-1, 3)$ , resulting in the overall allocation  $x^i = (1, 1, 1, 1)$ ,  $x^j = (3, 3, 3, 3)$ .

That allocation is not Pareto efficient, being dominated by  $x_\varepsilon^i = (1 - \varepsilon, 1 - \varepsilon, 1 + \varepsilon, 1 + \varepsilon)$ ,  $x_\varepsilon^j = (3 + \varepsilon, 3 + \varepsilon, 3 - \varepsilon, 3 - \varepsilon)$  with  $1/2 \geq \varepsilon > 0$ . In this case, sticky prices are incompatible with Pareto efficiency, which requires a change of relative prices  $\hat{p}_{11}/\hat{p}_{12}$  from 3 to 1.

4. In order to predict market-clearing prices at future date events, agents must know the distribution of endowments there, hence the quantities traded on today's asset markets. Correlatively, to verify the ex ante time-consistency of observed equilibria, it is not sufficient to specify only the standard primitive concepts like initial endowments, preferences, and technological constraints. Indeed, it is also necessary to specify previously held expectations concerning future prices, as possibly revealed by previous trading on asset markets.
5. Ruling out the time-inconsistent sequence  $(E_a, E_b)$  could be achieved in at least two ways:<sup>2</sup> by opening forward markets (more generally contingent markets) at period 0, thereby displacing the initial position of trades at period 1;<sup>3</sup> or by pre-determining the spot prices at which period 1 trades will occur. In economies with many commodities, stickiness of some

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<sup>2</sup>A third possibility, explored in Hahn (1999) and Chichilnisky (1999) - see also the appendix of Drèze (1999) - introduces sequential markets for options (on prices).

<sup>3</sup>See, however, Svensson (1976) on a potential shortcoming of forward markets under

prices could thus play the ancillary role of facilitating a smooth sequence of incomplete-markets equilibria. The example under comment 3 above reveals, however, that price stickiness will not help when a change in prices is required for time consistency.

6. Time-consistency of market equilibria is a relevant concern where there exists a continuum of equilibria. Examples are overlapping generations models (see Kehoe and Levine (1985)) and incomplete market models with nominal securities (see Balasko and Cass (1989) and Geanakoplos and Mas-Colell (1989)). Continua of equilibria arise also in economies with money (Drèze and Polemarchakis (2001)) or with price rigidities (Herings (1998) and Citanna et al. (2001)). The main concern, in such economies, is to alleviate direct inefficiencies (like uninsurable inflation uncertainty or underutilization of resources); time-consistency may then emerge as a by-product.

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rational expectations: if agents expect relative prices to be the same in tomorrow's spot markets as on today's forward markets, they have no incentives to trade on the latter.

## Relation to the Literature

1. Attention to time-inconsistent sequential equilibria is present in earlier work on incomplete markets (Hellwig (1983)) and on production economies with linear activities (Mandler (1995)) and on Sraffa's (1960) price theory (Mandler (2002)). This note shows that time-inconsistent sequential equilibria also arise in pure exchange economies with unique intertemporal equilibria.
2. Earlier work on potential inefficiencies associated with multiple equilibria includes work on sunspots, see Cass and Shell (1983) and on options, see Chichilnisky (1999). The tradition in the sunspots literature is to assume a complete set of markets for contingent claims. The inefficiencies are then related to limited participation of some agents (e.g. future generations) in the market trading at the initial date.<sup>4</sup> The work on options is concerned with the possibility of forcing efficiency through (sophisticated) successive rounds of trading. Our example does not involve limited participation, and stresses the counter-intuitive distinction between complete versus sequentially complete markets for contingent claims to commodities, in the absence of the successive rounds of trading apt to restore efficiency.
3. Common beliefs equilibria embody a requirement of coordination of expectations that remains stringent. More permissive equilibrium concepts are found in the rationalizability literature, introduced by Bernheim (1984) and Pearce (1984), and the closely related literature on expectational stability, see Lucas (1978) and DeCanio (1979). A recent overview of this literature can be found in Guesnerie (2002). In this literature, common knowledge of future price expectations is abandoned. Agents are allowed to hold any price expectations that can be rationalized, i.e. price expectations that would be compatible with optimizing behavior of other agents. Expectations of agents might be idiosyncratic.

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<sup>4</sup>See also footnote 3 above.

## Appendix 1

The equilibrium defined by  $p = (1/2, 1/2, 1/2, 1/2)$ ,  $x^i = x^j = (2, 2, 2, 2)$  is unique.

- i. For  $k = 0, 1$ , if  $\min(x_{k1}^i, x_{k2}^i) \leq 1$ , Pareto efficiency requires  $x_{k1}^i = x_{k2}^i$ ; if  $\min(x_{k1}^i, x_{k2}^i) \leq 3$ , Pareto efficiency requires  $x_{k1}^j = x_{k2}^j$ . But  $x_{kl}^i + x_{kl}^j = 4$ ,  $l = 1, 2$ ; hence, at least one of the above inequalities holds and  $x_{k1}^i = x_{k2}^i$ ,  $x_{k1}^j = x_{k2}^j$ .
- ii. It follows that  $u^i = (x_{01}^i x_{11}^i)^{\frac{1}{2}}$ ,  $u^j = (x_{01}^j x_{11}^j)^{\frac{1}{2}}$ , so that prices supporting a Pareto efficient allocation must verify  $p_{01} + p_{02} = p_{11} + p_{12}$ . At such prices, a competitive allocation with  $x_{k1}^i = x_{k2}^i$ ,  $k = 1, 2$ , has  $x_{01}^i = x_{02}^i = x_{11}^i = x_{12}^i$ , and similarly for  $x^j$ .
- iii. Suppose  $x_{01}^i = 1$ . From budget equality for agent  $i$ ,  $p_{01} + p_{02} + p_{11} + p_{12} = 2p_{01} + 2p_{02} + 4p_{12}$ . Since  $p_{01} + p_{02} = p_{11} + p_{12}$ , it follows that  $p_{12} = 0$ , which is at odds with utility maximization. Consequently,  $x_{01}^i \neq 1$ .

If  $x_{01}^i \neq 1$ , then the utility function of either agent  $i$  or agent  $j$  is differentiable at  $x^i$  or  $x^j$ , respectively. From the relevant marginal rate of substitution, we obtain that  $p_{01} = p_{02} = p_{11} = p_{12}$ , and  $x^i = x^j = (2, 2, 2, 2)$ . The net trades are  $z^i = (0, 0, 2, -2)$  and  $z^j = (0, 0, -2, 2)$ .

## Appendix 2

The example of Appendix 1 is expanded to allow for probabilistic expectations regarding period 1 prices. The equilibrium there will either be supported by prices  $(1/2, 1/2)$  or it will be at  $E_b$  with prices determined endogenously by the period 1 resources of the two agents. We limit ourselves to exhibit one REE. Within period efficiency remains, so  $x_{k1}^i = x_{k2}^i$  and  $x_{k1}^j = x_{k2}^j$ ,  $k = 1, 2$ .

Write  $\tilde{p}_{11}$  for the value of  $p_{11}$  in case the equilibrium is at  $E_b$ . Agent  $i$ 's net trade in bonds at period 0 is  $b^i = (2 - x_0^i)/q$ , where  $x_0^i = x_{01}^i = x_{02}^i$ . Accordingly,  $i$ 's resources in period 1 are worth  $(2 - x_0^i)/q + 4p_{12}$ ; this is also the quantity  $x_1^i = x_{11}^i = x_{12}^i$  that  $i$  can consume, given  $p_{11} + p_{12} = 1$ . Thus, his period 1 consumption will be either  $(2 - x_0^i)/q + 4(1 - \tilde{p}_{11})$  or  $(2 - x_0^i)/q + 2$ , with equal probabilities. His expected utility is

$$Eu^i = (x_0^i)^{\frac{1}{2}} \left[ \frac{1}{2} \left( \frac{2 - x_0^i}{q} + 4(1 - \tilde{p}_{11}) \right)^{\frac{1}{2}} + \frac{1}{2} \left( \frac{2 - x_0^i}{q} + 2 \right)^{\frac{1}{2}} \right].$$

The FOC may be written as

$$[2 - x_0^i + 4q(1 - \tilde{p}_{11})][2 - x_0^i + 2q] = (x_0^i)^2. \quad (0.1)$$

By the same reasoning, the FOC of agent  $j$  is

$$[2 - x_0^j + 4q\tilde{p}_{11}][2 - x_0^j + 2q] = (x_0^j)^2. \quad (0.2)$$

We wish to find values of  $q$  and  $\tilde{p}_{11}$  such that  $(2 - x_0^i)/q + 4(1 - \tilde{p}_{11}) = 1$  and  $(2 - x_0^j)/q + 4\tilde{p}_{11} = 3$ , implying equilibrium  $E_b$  at prices  $(\tilde{p}_{11}, 1 - \tilde{p}_{11})$ , as well as  $x_0^i + x_0^j = 4$ .

Straightforward substitutions lead to a quartic equation in  $q$  which has three negative roots and a single positive root, namely 1.064. Bond trades are  $b^i = -b^j = 0.37$ , and  $\tilde{p}_{11} = 0.837$ . Because commodities are not storable, the negative real rate implied by  $q > 1$  is acceptable.

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